Inverses

*If the graph of $f(x)$ passes the **horizontal line test**, then $f^{-1}(x)$ is a function.*

$f(x)=\frac{1}{5}x^3-2$

*$f(x)$ is said to be **one-to-one** if its inverse is also a function.*

Example 1 Graphs of $f(x)$ and $f^{-1}(x)$

If (x, y) is on the graph of $f(x)$, then (,) is on the graph of $f^{-1}(x)$.

A]

x	$f(x)$
-7	-31
-3	-9
	-15
5	
9	

x	$f^{-1}(x)$
-31	-7
-9	
	1
17	
33	9

The graphs of $f(x)$ and $f^{-1}(x)$ have _____ about the line _____.

B]

Example 2 Equations of $f(x)$ and $f^{-1}(x)$

To find the equation of the inverse of f(x):

1. Replace $f(x)$ with y
2. *Switch* x and y
3. *Solve for* y
4. Replace y with $f^{-1}(x)$

A] $f(x)=3x+9$

B] $f(x)=\frac{-5x+3}{2}$

C] $f(x)=\frac{1}{2}x+\frac{3}{4}$

Example 3 Verify that $f(x)$ and $g(x)$ are inverses

If $f(x)$ and $g(x)$ are **inverses**, then $(f\circ g)(x)=x$ and $(g\circ f)(x)=x$.

Use function composition to verify that $f(x)$ and $g(x)$ are inverses.

A] $f(x)=\frac{1}{4}x+6$ $\quad g(x)=4x-24$

B] $f(x)=10x+8$ $\quad g(x)=\frac{1}{10}x-\frac{4}{5}$

Example 4 When is $f(x)$ one-to-one?

If the graph of $f(x)$ passes the vertical line test, then it is a function.

If the graph of $f(x)$ passes the horizontal line test, then its inverse $f^{-1}(x)$ is a function.

If both $f(x)$ and $f^{-1}(x)$ are functions, then $f(x)$ is said to be **one-to-one**.

Use the graphs to decide which are one-to-one.

A]

B]

C]